

LOGARITHM & QUADRATIC EQUATION

Characteristic & Mantissa.

1) $\ln e = 1$ 2) $\log_{10} 2 = .3010$, (3) $\log_{10} 3 = .4771$

4) $\log_e 10 = \ln 10 = 2.303$ (5) $\ln 2 = .693$.

6) log of any No consists of 2 parts $\begin{matrix} \swarrow & \text{Integer} \\ \searrow & \text{fractional} \end{matrix}$

7) Integral Part is known as characteristic
Fraction Part is known as mantissa.

$\log_{10} 3 = 0.4771$
 (h. $\rightarrow$ Mantissa.)

$\text{Mantissa} \in [0, 1)$

$\log_{10} N = \alpha + \beta; \beta \in [0, 1)$

$\beta = \text{Mant.}$

$\alpha = \text{Charct}$

LOGARITHM & QUADRATIC EQUATION

$$x = [x] + \{x\}$$

$$\log_a N = \underbrace{[\log_a N]}_{(h)} + \underbrace{\{\log_a N\}}_{\text{Man.}}$$

Mantissa of any log is always +ve.

Ex:- 1) $\log_{10} 34.56 = 1.5386$

$(h = 1, \text{mant} = .5386)$

$$\begin{aligned} 2) \log_{10} 345.6 &= \log_{10} (34.56 \times 10) \\ &= \log_{10} 34.56 + \log_{10} 10 \\ &= 1.5386 + 1 = \boxed{2.5386} \end{aligned}$$

$$3) \log 3456 = \log_{10} (34.56 \times 10^2)$$

$$= \log_{10} 34.56 + 2$$

$$= 1.5386 + 2$$

$$\boxed{\log_{10} 3456 = 3.5386}$$

$\boxed{(h = 3)} \text{ mant} = .5386$

No of digits Before Decimal
 $= (h + 1)$

LOGARITHM & QUADRATIC EQUATION

$$\log_{10} 34.56 = 1.5386$$

$$\log_{10} 3.456 = 0.5386$$

$$\begin{array}{r} 33 \\ 4771 \\ \hline 23855 \end{array}$$

Q No. of digit in 3^{50}

$$\log_{10} 3^{50} = 50 \log_{10} 3$$

$$= 50 \times .4771$$

$$= 23.855$$

$$Ch = 23 \Rightarrow \text{No of digit} = 23 + 1 = 24$$

Q No of digit in 2^{100}

$$\log_{10} 2^{100}$$

$$100 \times \log_{10} 2$$

$$100 \times .3010$$

$$[30.10] = 30 = Ch$$

$$\begin{aligned} \text{No of digit} &= 30 + 1 \\ &= 31 \end{aligned}$$

LOGARITHM & QUADRATIC EQUATION

No of zeroes / Zero after decimal before 1st non Zero digit

$$\{5.4\} = .4$$

$$\{5\} = 0$$

$$\{-5.4\} = 1 - .4 = .6$$

$$= |h+1|$$

$$\log_{10} 34.56 = 1.5386$$

$$\log(\underline{.3456}) = \log(34.56 \times 10^{-2}) = -2 + 1.5386 =$$

$$= -0.4614$$

$$h = [-.4614] = -1$$

$$M_{am} = \{-.4614\} = 1 - .4614 = .5386$$

$$|-1+1| = 0$$

LOGARITHM & QUADRATIC EQUATION

$$\log \left(\underbrace{.00}_{2} 3456 \right) = \log (34.56 \times 10^{-4}) = -4 + 1.5386 \\ = -2.4614$$

$$(h = [-2.4614]) = -3.$$

$$M_{\text{am}} = \{-2.4614\} = .5386$$

No of zeroes after decimal Before non Zero Digit

$$= |-3+1| = |-2| = 2.$$

LOGARITHM & QUADRATIC EQUATION

Q Find No. of zeros in $\left(\frac{2}{3}\right)^{100}$

$$\log_{10}\left(\frac{2}{3}\right)^{100} = 100 \log_{10}\left(\frac{2}{3}\right) = 100 \times \{\log_{10} 2 - \log_{10} 3\}$$

$$= 100 \{0.3010 - 0.4771\} = 100 \times -0.1761$$

$$= -17.61 \rightarrow Ch = [-17.61] = -18$$

$$\text{No of zeros} = |-18 + 1| = 17$$

LOGARITHM & QUADRATIC EQUATION

R_k

$$\log_{10} 34.56 = 1.5386$$



$$10 \leq 34 < 100$$

$$10^1 \leq 34 < 10^2$$

$$\log_{10} N = 2. \sim$$

$$\log_{10} N = 3. \sim$$

If $ch = 2$ then

$$10^2 \leq N < 10^3$$

If $ch = 3$ then

$$10^3 \leq N < 10^4$$

LOGARITHM & QUADRATIC EQUATION

Q Given that $\log_3 N = \alpha + \beta$; $\beta \in [0, 1)$
 No find the No of Integral values of N

if $\alpha = 4$.

8, 9, 10, 11

11 - 8 + 1

4

$$\log_3 N = \alpha + \beta \rightarrow \text{part}$$

$$\alpha = 4 = (h)$$

$$[\log_3 N] = 4$$

$$4 \leq \log_3 N < 5$$

$$3^4 \leq N < 3^5$$

$$81 \leq N < 243$$

$$\{81, 82, 83, 84, \dots, 242\}$$

$$243 - 81 = 162$$

$$242 - 81 + 1$$

$$\text{Rk. } [x] = n$$

$$n \leq x < n+1$$

$$[x] = 7$$

$$7 \leq x < 8$$

$$[7.1] = 7$$

$$[7.4] = 7$$

$$[7] = 7$$

$$[7.99] = 7$$

LOGARITHM & QUADRATIC EQUATION

Q Given that $\log_5 N = \alpha + \beta$; $\beta \in [0, 1)$

Find the No of Integral values of N if $\alpha = 3$.

125, 126, 127

$$\begin{aligned} \alpha &= (h = 3) \\ h &= \lceil \log_5 N \rceil = 3 \\ 3 &\leq \log_5 N < 4 \\ 5^3 &\leq N < 5^4 \\ \boxed{125 \leq N < 625} \end{aligned}$$

Sunday — Sunday
8 Days

$$\underline{\underline{624 - 125 + 1}}$$

$$\begin{aligned} \text{No of Int} &= \frac{625 - 125}{1} \\ &= 500 \checkmark \end{aligned}$$

LOGARITHM & QUADRATIC EQUATION

Q Let $\log_3 N = \alpha_1 + B_1$, $\log_5 N = \alpha_2 + B_2$

; $B_1, B_2 \in [0, 1)$ Find No of Integral

values of N if $\alpha_1 = 4$ & $\alpha_2 = 2$

$$\begin{array}{r} 125 \\ 81 \\ \hline \end{array}$$

$$\begin{aligned} \alpha_1 &= 4 \\ \lfloor \log_3 N \rfloor &= 4 \\ 3^4 &\leq N < 3^5 \\ 81 &\leq N < 243 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= 2 \\ \lfloor \log_5 N \rfloor &= 2 \\ 2 &\leq \log_5 N < 3 \end{aligned}$$

$$\cap \quad 25 \leq N < 125$$

$$\begin{aligned} 81 &\leq N < 125 \quad N = 124 - 81 + 1 \\ &= 44 \end{aligned}$$

$$10^q - 10^{q-1} = 10^q \left(1 - \frac{1}{10}\right) = 9 \cdot 10^{q-1} \quad | \quad 10^{p+1} - 10^p = 10^p (10 - 1) = 9 \cdot 10^p$$

Q If P is the No of Integers whose log to Base 10 have $\text{ch} = p$ & Q the No.

of Integers the log of whose Reciprocal to Base 10 have $\text{ch} = -q$ Show that

$$\log_{10} P - \log_{10} Q = p - q + 1$$

$$10^q \geq Y > 10^{q-1} \Rightarrow \text{No of Int} = 10^q - 10^{q-1}$$

$$\log_{10} (10^{p+1} - 10^p) - \log_{10} (10^q - 10^{q-1}) = Q$$

$$\log_{10} (9 \cdot 10^p) - \log_{10} (9 \cdot 10^{q-1}) = p - (q-1)$$

$$[\log_{10} N] = p$$

$$p \leq \log_{10} N < p+1$$

$$10^p \leq N < 10^{p+1}$$

$$P = \text{No of Int} = 10^{p+1} - 10^p$$

$$[\log_{10} \frac{1}{Y}] = -q$$

$$-q \leq \log_{10} \frac{1}{Y} < -q+1$$

$$-q \leq -\log_{10} Y < -q+1$$

$$q \geq \log_{10} Y > q-1$$

Q. $x = 1 + \log_a bc$, $y = 1 + \log_b ca$, $z = 1 + \log_c ab$ then S.T. $xyz = xy + yz + zx$

$$= \log_a a + \log_a bc$$

$$x = \log_a abc$$

$$\frac{1}{x} = \log_{abc} a$$

$$y = \log_b b + \log_b ca$$

$$y = \log_b abc$$

$$\frac{1}{y} = \log_{abc} b$$

$$\frac{1}{z} = \log_{abc} c$$

$$\log_{abc} a + \log_{abc} b + \log_{abc} c =$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$\log_{abc} abc = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$= 1 = \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \Rightarrow xyz = xy + yz + zx$$

LOGARITHM & QUADRATIC EQUATION

1. The value of $\log_2 \left(\frac{1}{\cancel{7}^{\cancel{3}} \log_7 0.125} \right)$, is

(A) 1

(B) 2

(C) 3

(D) 4

$$\log_2 \left(\frac{\cancel{1000}^3}{\cancel{125}} \right) = 3.$$

$$\begin{aligned}\frac{1}{\sqrt{2}} &= \left(\frac{1}{2}\right)^{\frac{1}{2}} \\ &= (2^{-1})^{\frac{1}{2}} \\ &= 2^{-\frac{1}{2}}\end{aligned}$$

$$\log_{\frac{1}{6}} 2 \cdot \log_5 36 \cdot \log_{17} 125 \cdot \log_{\frac{1}{12}} 17$$

$$+ \frac{\cancel{\log 2}}{\cancel{\log \frac{1}{6}}} \times 2 \frac{\cancel{\log 36}}{\cancel{\log 5}} \times 3 \frac{\cancel{\log 125}}{\cancel{\log 17}} \times \frac{\log 17}{\frac{1}{2} \cancel{\log \frac{1}{12}}}$$

$$= 12$$